The Calculation of the Temperature Rise and Load Capability of Cable Systems

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In 1932 D. M. Simmons published a series of articles entitled, "Calculation of the Electrical Problems of Underground Cables." Over the intervening 25 years this work has achieved the status of a handbook on the subject. During this period, however, there have been numerous developments in the cable art, and much theoretical and experimental work has been done with a view to obtaining more accurate methods of evaluating the parameters involved. The advent of the pipe-type cable system has emphasized the desirability of a more rational method of calculating the performance of cables in order that a realistic comparison may be made between the two systems.

In this paper the authors have endeavored to extend the work of Simmons by presenting under one cover the basic principles involved, together with more recently developed procedures for handling such problems as the effect of the loading cycle and the temperature rise of cables in various types of duct structures. Included as well are expressions required in the evaluation of the basic parameters for certain specialized allied procedures. It is thought that a work of this type will be useful not only as a guide to engineers entering the field and as a reference to the more experienced, but particularly as a basis for setting up computation methods for the preparation of industry load capability and a-c/d-c ratio compilations.

The calculation of the temperature rise of cable systems under essentially steady-state conditions, which includes the effect of operation under a repetitive load cycle, as opposed to transient temperature rises due to the sudden application of large amounts of load, is a relatively simple procedure and involves only the application of the thermal equivalents of Ohm's and Kirchoff's Laws to a relatively simple thermal circuit. Because this circuit usually has a number of parallel paths with heat flows entering at several points, however, care must be exercised in the method used of expressing the heat flows and thermal resistances involved, and differing methods are used by various engineers. The method employed in this paper has been selected after careful con-

sideration as being the most consistent and most readily handled over the full scope of the problem.

All losses will be developed on the basis of watts per conductor foot. The heat flows and temperature rises due to dielectric loss and to current-produced losses will be treated separately, and, in the latter case, all heat flows will be expressed in terms of the current produced loss originating in one foot of conductor by means of multiplying factors which take into account the added losses in the sheath and conduit.

In general, all thermal resistances will be developed on the basis of the per conductor heat flow through them. In the case of underground cable systems, it is convenient to utilize an effective thermal resistance for the earth portion of the thermal circuit which includes the effect of the loading cycle and the mutual heating effect of the other cable of the system. All cables in the system will be considered to carry the same load currents and to be operating under the same load cycle.

The system of nomenclature employed is in accordance with that adopted by the Insulated Conductor Committee as standard, and differs appreciably from that used in many of the references. This system represents an attempt to utilize in so far as possible the various symbols appearing in the American Standards Association Standards for Electrical Quantities, Mechanics, Heat and Thermo-Dynamics, and Hydraulics, when these symbols can be used without ambiguity. Certain symbols which have long been used by cable engineers have been retained, even though they are in direct conflict with the above-mentioned standards.

Nomenclature

\( (AF) \) = attainment factor, per unit (pu)

\( A_s \) = cross-section area of a shielding tape or skid wire, square inches

\( a \) = thermal diffusivity, square inches per hour

\( C_f \) = conductor area, circular inches

\( d \) = distance, inches

\( d_{c1} \) etc. = from center of cable no. 1 to center of cable no. 2 etc.

\( d'_{c1} \) etc. = from center of cable no. 1 to image of cable no. 2 etc.

\( d_{c1} \) etc. = from center of cable no. 1 to a point of interference

\( d_{r1} \) etc. = from image of cable no. 1 to a point of interference

\( D \) = diameter, inches

\( D_k \) = inside of annular conductor

\( D_e \) = outside of conductor

\( D_i \) = outside of insulation

\( D_m \) = mean diameter of sheath

\( D_{e1} \) = effective (circumscribing circle) of several cables in contact

\( D_j \) = inside of duct wall, pipe or conduit

\( D_k \) = diameter at start of the earth portion of the thermal circuit

\( D_x \) = fictitious diameter at which the effect of loss factor commences

\( E \) = line to neutral voltage, kilovolts (kV)

\( \epsilon \) = coefficient of surface emissivity

\( \epsilon \) = specific inductive capacitance of insulation

\( f \) = frequency, cycles per second

\( F_{int} \) = products of ratios of distances

\( F(x) \) = derived Bessel function of x (Table III and Fig. 1)

\( G \) = geometric factor

\( G_1 \) = applying to insulation resistance (Fig. 2 of reference 1)

\( G_2 \) = applying to dielectric loss (Fig. 2 of reference 1)

\( G_9 \) = applying to a duct bank (Fig. 2)

\( l \) = conductor current, kiloamperes

\( k \) = skin effect correction factor for annular and segmental conductors

\( k_p \) = relative transverse conductivity factor for calculating conductor proximity effect

\( L \) = lay of a shielding tape or skid wire, inches

\( L_s \) = depth of reference cable below earth's surface, inches

\( L_b \) = depth to center of a duct bank (or backfill), inches

\( (L) \) = load factor, per unit

\( (L_F) \) = loss factor, per unit

\( n \) = number of conductors per cable

\( n' \) = number of conductors within a stated diameter

\( N \) = number of cables or cable groups in a system

\( P \) = perimeter of a duct bank or backfill, inches

\( \phi \) = power factor of the insulation

\( q_{be} \) = ratio of the sum of the losses in the conductors and sheaths to the losses in the conductors

\( q_{be} \) = ratio of the sum of the losses in the conductors, sheath and conduit to the losses in the conductors

\( R \) = electrical resistance, ohms

\( R_{d1} \) = d-c resistance of conductor

\( R_{sl} \) = total a-c, d-c resistance of conductor

\( R_s \) = d-c resistance of sheath or of the parallel paths in a shield-skid wire assembly

\( R_b \) = thermal resistance (per conductor losses) or thermal ohm-feet

\( R_{e1} \) = of insulation

\( R_{k1} \) = of jacket

\( R_{ed} \) = between cable surface and surrounding enclosure

Paper 57-660, recommended by the AIEE Insulated Conductor Committee and approved by the AIEE Technical Operations Department for presentation at the AIEE Summer General Meeting, Montreal, Que., Canada, June 24-28, 1957. Manuscript submitted March 20, 1957; made available for printing April 18, 1957.

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\( R_d \) = of duct wall or asphalt mastic covering
\( R_{se} \) = total between sheath and diameter
\( D_s \), including each \( R_j \), \( R_{se} \) and \( R_d \)
\( R_{oe} \) = between conduit and ambient
\( R_{re} \) = effective between diameter \( D_s \) and ambient earth including the effects of loss factor and mutual heating by other cables
\( R_{re} \) = effective between conductor and ambient for conductor loss
\( R_{re} \) = effective transient thermal resistance of cable system
\( R_{re} \) = effective between conductor and ambient for dielectric loss
\( R_{re} \) = of the interference effect
\( R_{re} \) = between a steam pipe and ambient earth
\( p \) = electrical resistivity, circular mil ohms per foot
\( \sigma \) = thermal resistivity, degrees centigrade centimeters per watt
\( s \) = distance in a 3-conductor cable between the effective current center of the conductor and the axis of the cable, inches
\( S \) = axial spacing between adjacent cables, inches
\( l, T \) = thickness (as indicated), inches
\( T \) = temperature, degrees centigrade
\( T_r \) = of ambient air or earth
\( T_c \) = of conductor
\( T_m \) = mean temperature of medium
\( \Delta T \) = temperature rise, degrees centigrade
\( \Delta T_{re} \) = of conductor due to current produced losses
\( \Delta T_{re} \) = of conductor due to dielectric loss
\( \Delta T_{re} \) = of a cable due to extraneous heat source
\( r \) = inferred temperature of zero resistance, degrees centigrade (C) (used in correcting \( R_{re} \) and \( R_d \) to temperatures other than 20 C)
\( V_w \) = wind velocity, miles per hour
\( W \) = losses developed in a cable, watts per conductor foot

\( W_e \) = portion developed in the conductor
\( W_s \) = portion developed in the sheath or shield
\( W_p \) = portion developed in the pipe or conduit
\( W_d \) = portion developed in the dielectric
\( X_m \) = mutual reactance, conductor to sheath or shield, microhms per foot
\( Y \) = the increment of a-c/d-c ratio, pu
\( Y_e \) = due to losses originating in the conductor, having components \( Y_{re} \) due to skin effect and \( Y_{re} \) due to proximity effect
\( Y_s \) = due to losses originating in the sheath or shield, having components \( Y_{re} \) due to circulating current effect and \( Y_{re} \) due to eddy current effect
\( Y_p \) = due to losses originating in the pipe or conduit
\( Y_a \) = due to losses originating in the armor

Thus

\[ T_c - T_a = \Delta T_c + \Delta T_d \]  

degrees centigrade

Each of these component temperature rises may be considered as the result of a rate of heat flow expressed in watts per foot through a thermal resistance expressed in thermal ohm feet (degrees centigrade feet per watt); in other words, the radial rise in degrees centigrade for a heat flow of one watt uniformly distributed over a conductor length of one foot.

Since the losses occur at several positions in the cable system, the heat flow in the thermal circuit will increase in steps. It is convenient to express all heat flows in terms of the loss per foot of conductor, and thus

\[ \Delta T_c = W_c (R_{re} + q_s R_{se} + q_r R_d) \]

degrees centigrade

in which \( W_c \) represents the losses in one conductor and \( R_d \) is the thermal resistance of the insulation, \( q_s \) is the ratio of the sum of the losses in the conductors and sheath to the losses in the conductors, \( R_{re} \) is the total thermal resistance between sheath and conduit, \( R_{se} \) is the ratio of the sum of the losses in conductors, sheath and conduit, to the conductor losses, and \( R_d \).
is the thermal resistance between the conduit and ambient.

In practice, the load carried by a cable is rarely constant and varies according to a daily load cycle having a load factor \((Lf)\). Hence, the losses in the cable will vary according to the corresponding daily load cycle having a loss factor \((Lf)\). From an examination of a large number of load cycles and their corresponding load and loss factors, the following general relationship between load factor and loss factor has been found to exist.\(^1\)

\[
(LF) = 0.3 (Lf) + 0.7 (Lf)^2 \text{ per unit} \quad (3)
\]

In order to determine the maximum temperature rise attained by a buried cable system under a repeated daily load cycle, the losses and resultant heat flows are calculated on the basis of the maximum load (usually taken as the average current for that hour of the daily load cycle during which the average current is the highest, i.e. the daily maximum one-hour average load) on which the loss factor is based and the heat flow in the last part of the earth portion of the thermal circuit is reduced by the factor \((LF)\). If this reduction is considered to start at a point in the earth corresponding to the diameter \(D_e\), equation 2 becomes

\[
\Delta T_e = W_e (R_t + g_e R_{sa} + g_e (R_{es} + (LF) R_{se})) \text{ degrees centigrade} \quad (4)
\]

In effect this means that the temperature rise from conductor to conduit to earth is made to depend on the heat loss corresponding to the maximum load whereas the temperature rise from diameter \(D_e\) to ambient is made to depend on the average loss over a 24-hour period. Studies indicate that the procedure of assuming a fictitious critical diameter \(D_c\) at which an abrupt change occurs in loss factor from 100% to actual gives results which vary closely approximate those obtained by rigorous transient analysis. For cables or duct in air where the thermal storage capacity of the system is relatively small, the maximum temperature rise is based upon the heat flow corresponding to maximum load without reduction of any part of the thermal circuit.

When a number of cables are installed close together in the earth or in a duct bank, each cable will have a heating effect upon all of the others. In calculating the temperature rise of any one cable, it is convenient to handle the heating effects of the other cables of the system by suitably modifying the last term of equation 4. This is permissible since it is assumed that all the cables are carrying equal currents, and are operating on the same load cycle. Thus for an \(N\)-cable system

\[
\Delta T_e = W_e (R_t + g_e R_{sa} + g_e (R_{es} + (LF) R_{se}))
\]

where the term in parentheses is indicated by the effective thermal resistance \(R_{se}'\).

The temperature rise due to dielectric loss is a relatively small part of the total temperature rise of cable systems operating at the lower voltages, but at higher voltages it constitutes an appreciable part and must be considered. Although the dielectric losses are distributed throughout the insulation, it may be shown that for single conductor cable and multicore shielded cable with round conductors the correct temperature rise is obtained by considering for transient and steady state that all of the dielectric loss \(W_d\) occurs at the middle of the thermal resistance between conductor and sheath or alternatly for steady-state conditions alone that the temperature rise between conductor and sheath for a given loss in the dielectric is half as much as if that loss were in the conductor. In the case of multicore conductor cables, however, the conductors are taken as the source of the dielectric loss.\(^1\)

The resulting temperature rise due to dielectric loss \(\Delta T_d\) may be expressed

\[
\Delta T_d = W_d (R_{se} + g_e R_{sa})' \text{ degrees centigrade} \quad (6)
\]

in which the effective thermal resistance \(R_{se}'\) is based on \(R_t, R_{sa}, \text{ and } R_{se}'\) (at unity loss factor) according to the particular case. The temperature rise at points in the cable system other than at the conductor may be determined readily from the foregoing relationships.

THE CALCULATION OF LOAD CAPABILITY

In many cases the permissible maximum temperature of the conductor is fixed and the magnitude of the conductor current (load capability) required to produce this temperature is desired. Equation 5(A) may be written in the form

\[
\Delta T_e = I^2 R_{dc} (1 + Y_e) R_{se}' \text{ degrees centigrade} \quad (7)
\]

in which the quantity \(R_{dc} (1 + Y_e)\) which will be evaluated later represents the effective electrical resistance of the conductor in microhms per foot, and which when multiplied by \(I (I \text{ in kiloamperes})\) will equal the loss \(W_e\) in watts per conductor foot actually generated in the conductor; \(R_{se}'\) is the effective thermal resistance of the thermal circuit.

\[
R_{se}' = R_t + g_e R_{sa} + g_e R_{se}' \text{ thermal ohm-feet} \quad (8)
\]

From equation 1 it follows that

\[
I = \sqrt{\frac{T_e - \Delta T_d}{R_{dc} (1 + Y_e) R_{se}'}} \text{ kiloamperes} \quad (9)
\]

\[\text{Reactor Material} & \quad \text{Circular Mil} & \quad \text{Ohms per Foot} \\
\text{Copper (100% IACS*)} & \quad 10.37 \ldots 234.5 \\
\text{Aluminum (61% IACS)} & \quad 17.00 \ldots 228.1 \\
\text{Commercial Bronze (43.6% IACS)} & \quad 23.8 \ldots 584.0 \\
\text{Copper (90 Cu-10 Zn)} & \quad 38.0 \ldots 912 \\
\text{Brass (78.9% IACS)} & \quad 70 Cu-30 Zn) & \quad 132.3 \ldots 236 \\
\text{Lead (70.5% IACS* )} & \quad * \text{International Annealed Copper Standard.}
\]

Calculation of Losses and Associated Parameters

**CALCULATION OF D-C Resistances**

The resistance of the conductor may be determined from the following expressions which include a lay factor of 2%; see Table 1.

\[
R_{dc} = \frac{1.02g_e}{CI} \text{ microhms per foot at 20 C} \quad (10)
\]

\[
= \frac{12.9}{CI} \text{ for 100% IACS copper} \quad (10A)
\]

\[
= \frac{21.2}{CI} \text{ for 61% IACS aluminum at 75 C} \quad (10B)
\]

where \(CI\) represents the conductor size in circular inches and where \(g_e\) represents the electrical resistivity in circular mil ohms per foot. To determine the value of resistance at temperature \(T\) multiply the resistance at 20 C by \((r+T)/(r+20)\) where \(r\) is the inferred temperature of zero resistance.

The resistance of the sheath is given by the expressions

\[
R_s = \frac{\rho_s}{4D_{sm}} \text{ microhms per foot at 20 C} \quad (11)
\]

\[
= \frac{37.5}{D_{sm}} \text{ for lead at 50 C} \quad (11A)
\]

\[
= \frac{4.75}{D_{sm}} \text{ for 61% aluminum at 50 C} \quad (11B)
\]

\[
D_{sm} = D_c - 1 \text{ inches} \quad (12)
\]

The resistance of intercalated shields or skin wires may be determined from the expression

\[
R_{s1} \text{ per path} = \frac{\pi \rho_s}{4A_s} \left(1 + \frac{\pi D_{sm}}{I}\right)^2 \text{ microhms per foot at 20 C} \quad (13)
\]

where \(A_s\) is the cross-section area of the
Table II. Recommended Values of $k_a$ and $k_p$

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<tr>
<th>Conductor Construction</th>
<th>Coating on Strands</th>
<th>Treatment</th>
<th>$k_a$</th>
<th>$k_p$</th>
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</table>

Notes:
1. The term “treated” denotes a completed conductor which has been subjected to a drying process similar to that employed on paper power cable.
2. Proximity effect on compact sector conductors may be taken as one-half of that for compact round having the same cross-sectional area and insulation thickness.
3. Proximity effect on annular conductors may be approximated by using the value for a concentric round conductor of the same cross-sectional area and spacing. The increased diameter of the annular type and the removal of metal from the center decreases the skin effect but, for a given axial spacing, tends to result in a decrease in proximity effect.
4. The values listed above for compact segmental refer to four segment constructions. The “uncoated-treated” values may also be taken as applicable to four segment compact, seg. 40, with an approximate 0.75 inch clearance. For “uncoated-treated” six segment hollow core compact segmental limited test data indicates $k_a$ and $k_p$ values of 0.29 and 0.23 respectively.

Table III. Skin Effect in % in Solid Round Conductor and in Conventional Round Concentric Strand Conductors

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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

The skin effect may be determined from the skin effect function $F(x)$

$$F(x) = \frac{x_1}{\sqrt[4]{k_2}}$$

where $x_1 = 0.875 \sqrt[4]{k_2}$ at 60 cycles.

The factor $k_2$ depends upon the conductor construction. For solid or conventional conductors appropriate values of $k_2$ will be found in Table II. The function $F(x)$ may be obtained from Table III or from the curves of Fig. 1 in terms of the ratio $R_{dc}/k_2$ at 60 cycles.

For annular conductors

$$k_2 = \frac{D_a - D_e}{D_e + D_a}$$

where $D_a$ and $D_e$ represent the outer diameter of the annular conductor. With comparison to the rigorous Bessel function solution for the skin effect in an isolated tubular conductor, it has been found that the 60-cycle skin effect of annular conductor when computed by equation 23 will not be in error by more than 0.01 in absolute magnitude for copper or aluminum IPCEA (Insulated Power Cable Engineers Association) filled...
Table IV. Mutual Reactance at 60 Cycles, Conductor to Sheath (or Shield)

<table>
<thead>
<tr>
<th>Dm/2S</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.4</td>
<td>21.1</td>
<td>20.5</td>
<td>19.9</td>
<td>19.4</td>
<td>18.9</td>
<td>18.3</td>
<td>17.8</td>
<td>17.4</td>
<td>16.9</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>27.7</td>
<td>26.9</td>
<td>26.2</td>
<td>25.5</td>
<td>24.8</td>
<td>24.1</td>
<td>23.5</td>
<td>23.9</td>
<td>22.3</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>57.0</td>
<td>39.5</td>
<td>38.4</td>
<td>37.8</td>
<td>37.2</td>
<td>36.9</td>
<td>36.4</td>
<td>36.1</td>
<td>35.9</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>52.9</td>
<td>50.7</td>
<td>48.7</td>
<td>46.9</td>
<td>45.2</td>
<td>43.6</td>
<td>42.1</td>
<td>40.7</td>
<td>39.4</td>
</tr>
</tbody>
</table>

Core conductors up through 5.0 CI and for hollow core concentrically stranded copper or aluminum oil-filled cable conductors up through 4.0 CI.

For values of x_p below 3.5, a range which appear to cover most cases of practical interest at power frequencies, the conductor proximity effect for cables in equilateral triangular formation in the same or in separate ducts may be calculated from the following equation based on an approximate expression given by Arnold (equation 7) for a system of three homogeneous, straight, parallel, solid conductors of circular cross section arranged in equilateral formation and carrying balanced 3-phase current remote from all other conductors or conducting material. The empirical transverse conductor factor k_p is introduced to make the expression applicable to stranded conductors. Experimental results suggest the values of k_p shown in Table II.

\[ Y_{ep} = F(x_p) \left( \frac{D_s}{S} \right)^2 \times \left[ \frac{1.18}{F(x_p) + 0.27} + 0.312 \left( \frac{D_s}{S} \right)^{1/4} \right] \]  

(24)

\[ x_p = \frac{6.80}{\sqrt{R_d/k_p}} \text{ at 60 cycles} \]  

(25)

When the second term in the brackets is small with respect to the first term as it usually is, equation 24 may be written

\[ Y_{ep} = 4F(x_p) \left( \frac{0.295(D_s/S)^{1/2}}{F(x_p) + 0.27} \right) \]  

(24A)

where the function F(x_p) is shown in Fig. 1.

The average proximity effect for conductors in cradle configuration in the same duct or in separate ducts in a formation approximating a regular polygon may also be estimated from equation 24 and 24(A). In such cases, S should be taken as the axial spacing between adjacent conductors.

The factor Y_s is the sum of two factors, Y_sc due to circulating current effect and Y_e due to eddy current effects.

\[ W_e = R_d (Y_s + Y_{se}) \]  

watts per conductor foot (26)

Because of the large sheath losses which result from short-circuited sheath operation with appreciable separation between metallic sheathed single conductor cables, this mode of operation is usually restricted to triplex cable or three single-conductor cables contained in the same duct. The circulating current effect in three metallic sheathed single-conductor cables arranged in equilateral configuration is given by

\[ Y_{se} = \frac{R_s/R_{se}}{1 + (X_m/R_s)^2} \]  

(27)

When \( (R_s/X_m)^2 \) is large with respect to unity as usually is the case of shielded non-leaded cables, equation 27 reduces to

\[ Y_{se} \approx \frac{X_m}{R_s/R_{se}} \text{ approximately} \]  

(27A)

\[ X_m = 0.882f \log \frac{2S}{D_{em}} \text{ microhms per foot} \]  

(28)

\[ X_m = 52.9 \log \frac{2S}{D_{em}} \text{ microhms per foot at 60 cycles} \]  

(28A)

where S is the axial spacing of adjacent cables. For a cradled configuration X_m may be approximated from

\[ X_m = 52.9 \log \left( \frac{2S}{D_{em}} \right) \sqrt{1 - \left( \frac{S}{D_p - S} \right)^{1/2}} \text{ microhms per foot at 60 cycles} \]  

(29)

\[ X_m = 52.9 \log \left( \frac{2S}{D_{em}} \right) \text{ approximately} \]  

(29A)

Table IV provides a convenient means for determining X_m for cables in equilateral configuration.

The eddy-current effect for single-conductor cables in equilateral configuration with open-circuited sheaths is

\[ Y_{se} = \frac{3R_s/R_{se}}{f} \left[ \frac{25}{D_{em}} \right] \left[ \frac{1 + 5}{12 \left( \frac{D_{em}}{2S} \right)^{1/2}} \right] \]  

(30)

\[ \left( \frac{2S}{D_{em}} \right) \] as in the case of lead sheaths.

When the sheaths are short-circuited, the sheath eddy loss will be reduced and may be approximated by multiplying equations 30 and 30(A) by the ratio \( R_s/(R_s^2 + X_m^2) \).

In computing average eddy current for cradled configuration, S should be taken equal to the axial spacing and not to the geometric-mean spacing. Equations 30 and 30(A) may be used to compute the eddy-current effect for single-conductor cables installed in separate ducts. Strictly speaking, these equations apply only to three cables in equilateral configuration but can be used to estimate losses in large cable groups when latter are so oriented as to approximate a regular polygon.

The eddy-current effect for a 3-conductor cable is given by Arnold:

\[ Y_{se} = \frac{3R_s/R_{se}}{f} \left[ \frac{(2S/D_{em})^3}{6.2R_s/(f)^2 + 1} \right] + \frac{(2S/D_{em})^4}{16(5.2R_s/(f)^2 + 1)} \]  

(31)

When \( (5.2R_s/f)^2 \) is large with respect to unity,

\[ Y_{se} = \frac{396}{R_s/R_{se}} \left( \frac{2S}{D_{em}} \right)^2 \text{ approximately at 60 cycles \( (31A) \)} \]

\[ s = 1.155T + 0.60 \times \text{the V gauge depth for compact sectors} \]  

\[ = 1.155T + 0.58 D_c \text{ for round conductors} \]  

(32)

\[ T \] is the insulation thickness, including thickness of shielding tapes, if any. While equation 31(A) will suffice for lead sheath cables, equation 31 should be used for aluminum sheaths.

On 3-conductor shielded paper lead cable it is customary to employ a 3- or 5-mil copper tape or bronze tape intercalated with a paper tape for shielding and binder purposes. The lineal d-c resistance of a copper tape 5 mils by 0.75 inch is about 2,200 microhms per foot of tape at 20 °C. The d-c resistance per foot of cable will be equal to the lineal resistance of the tape multiplied by the lay correction factor as given by the expansion expression for the square-root sign in equation 13. In practice the lay correction factor may vary from 4 to 12 or more resulting in shielding and binder assembly resistances.

Table V. Specific Inductive Capacitance of Insulations

<table>
<thead>
<tr>
<th>Material</th>
<th>( \epsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polyethylene, . . . . . . . . .</td>
<td>2.3</td>
</tr>
<tr>
<td>Paper insulation (solid types)</td>
<td>3.7</td>
</tr>
<tr>
<td>Paper insulation (other types)</td>
<td>3.3-4.2</td>
</tr>
<tr>
<td>Rubber and rubber-like compounds</td>
<td>5.5</td>
</tr>
<tr>
<td>Varnished cambric</td>
<td>5.5</td>
</tr>
</tbody>
</table>
ances of approximately 10,000 or more microhms per foot of cable. Even on the assumption that the assembly resistance is halved because of contact with adjacent conductors and the lead sheath computations made using equations 27 and 30 show that the resulting circulating and eddy current losses are a fraction of 1% on sizes of practical interest. For this reason it is customary to assume that the losses in the shielding and binder tapes of 3-conductor shielded paper lead cable are negligible. In cases of nonlead rubber power cables where lapped metallic tapes are frequently employed, tube effects may be present and may materially lower the resistance of the shielding assembly and hence increase the losses to a point where they are of practical significance.

An exact determination of the pipe loss effect \( Y_p \) in the case of single-conductor cables installed in nonmagnetic conduit or pipe is a rather involved procedure as indicated in reference 7. Equation 31 may be used to obtain a rough estimate of \( Y_p \) for cables in cradled formation on the bottom of a nonmagnetic pipe, however by taking the average of the results obtained for wide triangular spacing with \( s = (D_p - D_0)/2 \) and for close triangular spacing at the center of the pipe with \( s = 0.578 D_0 \). The mean diameter of the pipe and its resistance per foot should be substituted for \( D_m \) and \( R \), respectively.

For magnetic pipes or conduit the following empirical relationships\(^*\) may be employed

\[
Y_p = \frac{1.54s - 0.115D_p}{R} \quad \text{(33)}
\]

\[
Y_p = \frac{0.89s - 0.115D_p}{R} \quad \text{(34)}
\]

\[
Y_p = \frac{0.34s + 0.175D_p}{R} \quad \text{(35)}
\]

These expressions apply to steel pipe\(^*\) and should be multiplied by 0.8 for iron conduit.\(^*\)

The expressions given for \( Y_x \) and \( Y_t \) above should be multiplied by 1.7 to find the corresponding in-pipe effects for magnetic pipe or conduit for both triangular and cradled configurations.

**Calculation of Dielectric Loss**

The dielectric loss \( W_d \) for 3-conductor shielded and single-conductor cable is given by the expression

\[
W_d = \frac{0.00276E^2 \cos \phi}{\log (27 + D_t)/D_c} \quad \text{watts per} \quad \text{conductor foot at 60 cycles} \quad \text{(36)}
\]

and for 3-conductor belted cable by

\[
W_d = \frac{0.019E^2 \cos \phi}{G_s} \quad \text{watts per} \quad \text{conductor foot at 60 cycles} \quad \text{(37)}
\]

where \( E \) is the phase to neutral voltage in kilovolts, \( \epsilon \) is the specific inductive capacitance of the insulation (Table V) \( T \) is its thickness and \( \cos \phi \) is its power factor. The geometric factor \( G_s \) may be found from Fig. 2 of reference 1.

For compact sector conductors the dielectric loss may be taken to that for a concentric round conductor having the same cross-sectional area and insulation thickness.

**Calculation of Thermal Resistance**

**Thermal Resistance of the Insulation**

For a single conductor cable,

\[
\bar{R}_t = 0.0125 \beta_1 \log D_t/D_c \quad \text{thermal ohm-feet} \quad \text{(38)}
\]

where \( \beta_1 \) is the thermal resistivity of the insulation (Table VI) and \( D_t \) is its diameter. In multiconductor cables there is a multipath heat flow between the conductor and sheath. The following expression\(^*\) represents an equivalent value which, when multiplied by the heat flow from one conductor, will produce the actual temperature elevation of the conductor above the sheath.

\[
\bar{R}_t = 0.00522 \beta_1 G_1 \quad \text{thermal ohm-feet} \quad \text{(39)}
\]

Values of the geometric factor \( G_1 \) for 3-conductor belted and shielded cables are given in Fig. 2 and Table VIII respectively of reference 1. On large size sector conductors with relatively thin insulation walls (i.e. ratios of insulation thickness to conductor diameter of the order of 0.2 or less); values of \( G_t \) for 3-conductor shielded cable as determined by back calculation, on the basis of an assumed insulation resistivity, from laboratory heat-run temperature-rise data, have not always confirmed theoretical values, and, in some cases, have yielded \( G_t \) values which approach those for a nonshielded, nonbelted construction.

### Table VI. Thermal Resistivity of Various Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>( \bar{\beta} ), C( \text{m}/\text{Watt} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper insulation (solid type)</td>
<td>700</td>
</tr>
<tr>
<td>Variscor bing (ammonium)</td>
<td>600</td>
</tr>
<tr>
<td>Paper insulation (other types)</td>
<td>500</td>
</tr>
<tr>
<td>Rubber and rubber-like</td>
<td>500</td>
</tr>
<tr>
<td>Jute and textile protective covering</td>
<td>400</td>
</tr>
<tr>
<td>Fiber duct</td>
<td>480</td>
</tr>
<tr>
<td>Polyethylene</td>
<td>450</td>
</tr>
<tr>
<td>Transite duct</td>
<td>200</td>
</tr>
<tr>
<td>Somatic</td>
<td>100</td>
</tr>
<tr>
<td>Concrete</td>
<td>85</td>
</tr>
</tbody>
</table>

**Thermal Resistance of Jackets, Duct Walls, and Somastic Coatings**

The equivalent thermal resistance of a jacket of paper, rubber or duct walls may be determined from the expression

\[
\bar{R} = 0.0104n(\frac{t}{D - t}) \quad \text{thermal ohm-feet} \quad \text{(40)}
\]

with appropriate subscripts applied to \( \bar{R} \), \( \beta \), and \( D \) in which \( D \) represents the outside diameter of the section and \( t \) its thickness. \( n \) is the number of conductors contained with the section contributing to the heat flow through it.

**Thermal Resistance Between Cable Surface and Surrounding Pipe, Conduit, or Duct Wall**

Theoretical expressions for the thermal resistance between a cable surface and a surrounding enclosure are given in reference 10. As indicated in Appendix I, these have been simplified to the general form

\[
\bar{R}_s = \frac{n' A}{1 + (B + CT_m)D'_t} \quad \text{thermal ohm-feet} \quad \text{(41)}
\]

in which \( A, B, \) and \( C \) are constants, \( D_t \) represents the equivalent diameter of the cable or group of cables and \( n' \) the number of conductors contained within \( D_t \). \( T_m \) is the mean temperature of the intervening medium. The constants \( A, B, \) and \( C \)

| Table VII. Constants for Use in Equations 41 and 41(A) |
|-----------------|--------|--------|--------|--------|
| Condition       | \( A' \) | \( B' \) | \( A'' \) | \( B'' \) |
| In metal conduit | 17     | 2.6    | 0.029  | 3.2    | 0.19   |
| In fiber duct in air | 17     | 2.1    | 0.016  | 5.6    | 0.33   |
| In fiber duct in concrete | 17     | 2.3    | 0.024  | 4.6    | 0.27   |
| In transite duct in air | 17     | 2.9    | 0.014  | 4.4    | 0.20   |
| In transite duct in concrete | 17     | 2.9    | 0.029  | 3.7    | 0.22   |
| Gas-filled pipe at 300 psi | 3.1    | 1.16   | 0.0033 | 2.1    | 0.61   |
| Oil-filled pipe | 0.84   | 0      | 0.0055 | 2.1    | 2.45   |

\( D_t = 1.00 \times \text{diameter of cable for one cable} \)

\( 1.65 \times \text{diameter of cable for two cables} \)

\( 2.15 \times \text{diameter of cable for three cables} \)

\( 2.60 \times \text{diameter of cable for four cables} \)
given in Table VII have been determined from the experimental data given in references 10 and 11.

If representative values of $T_n = 60^\circ C$ are assumed, equation 41 reduces to

$$ R_{ed} = \frac{n' A'}{D_{d}^t + B'} \text{ thermal ohm-feet} \hspace{1cm} (41A) $$

It should be noted that in the case of ducts, $R_{ed}$ is calculated to the inside of the duct wall and the thermal resistance of the duct wall should be added to obtain $R_e$.

**Thermal Resistance from Cables, Conductors, or Ducts Suspended in Air**

The thermal resistance $R_e$ between cables, conduits, or ducts suspended in still air may be determined from the following expression which is developed in Appendix I.

$$ R_e = \frac{15.6 n'}{D_{d}^t \left[ (\Delta T / D_{d}^t) / 4 + 1.6 \left( 1 + 0.0157 T_n \right) \right]} \text{ thermal ohm-feet} \hspace{1cm} (42) $$

In this equation $\Delta T$ represents the difference between the cable surface temperature $T_s$ and ambient air temperature $T_a$ in degrees centigrade, $T_n$ the average of these temperatures and $n'$ the coefficient of emissivity of the cable surface. Assuming representative values of $T_s = 60$ and $T_a = 50^\circ C$, and a range in $D_{d}^t$ of from 2 to 10 inches, equation 42 may be simplified to

$$ R_e = \frac{9.5 n'}{1 + 1.7 D_{d}^t (\epsilon + 0.41)} \text{ thermal ohm-feet} \hspace{1cm} (42A) $$

The value of $\epsilon$ may be taken as equal to 0.95 for pipes, conduits or ducts, and painted or braided surfaces, and from 0.2 to 0.5 for lead and aluminum sheaths, depending upon whether the surface is bright or corroded. It is interesting to note that equation 42(A) checks the IPCEA method of determining $R_e$ very closely with $\epsilon = 0.41$ for diameters up to 3.5 inches. In the IPCEA method $R_e = 0.00411 n' B / D_{d}^t$ where $B = 650 + 314 D_{d}^t$ for $D_{d}^t = 0 - 1.75$ inches and $B = 1,200$ for larger values of $D_{d}^t$.

**Effective Thermal Resistance Between Cables, Ducts, or Pipes, and Ambient Earth**

As previously indicated, an effective thermal resistance $R_e'$ may be employed to represent the earth portion of the thermal circuit in the case of buried cable systems. This effective thermal resistance includes the effect of loss factor and, in the case of a multicable installation, also the mutual heating effects of the other cables of the system. In the case of cables in a concrete duct bank, it is desirable to further recognize a difference between the thermal resistivity of the concrete $\beta_c$ and the thermal resistivity of the surrounding earth $\beta_e$.

The thermal resistance between any point in the earth surrounding a buried cable and ambient earth is given by the expression\(^\dagger\)

$$ R_{ps} = 0.012 \beta_e \log d'/d \text{ thermal ohm-feet} \hspace{1cm} (43) $$

in which $\beta_e$ is the thermal resistivity of the earth, $d'$ is the distance from the image of the cable to the point $P$, and $d$ is the distance from the cable center to $P$. From this equation and the principles discussed in references 3, 12, and 13, the following expressions may be developed, applicable to directly buried cables and to pipe-type cables.

$$ R_e' = 0.012 \beta_e n' \times \left[ \log \frac{D_s^t}{D_s^t + (LF) \log \left( \frac{4L}{D_s^t} F \right)} \right] + \frac{0.012 (\beta_e - \beta_n) n' (LF) G_0}{D_s^t} \text{ thermal ohm-feet} \hspace{1cm} (44A) $$

The geometric factor $G_0$, as developed in Appendix II is a function of the depth to the center of the concrete enclosure $L_s$ and its perimeter $P$, and may be found conveniently from Fig. 2 in terms of the ratio $L_s^2 / P$ and the ratio of the longest to short dimension of the enclosure.

For buried cable systems $T_s$ should be taken as the ambient temperature at the depth of the hottest cable. As indicated in reference 12, the expressions used throughout this paper for the thermal resistance and temperature rise of buried cable systems are based on the hypothesis suggested by Kennelly applied in accordance with the principle of superposition. According to this hypothesis, the isothermal-heat flow field and temperature rise at any point in the soil surrounding a buried cable can be represented by the steady-state solution for the heat flow between two parallel cylinders (constituting a heat source and sink) located in a vertical plane in an infinite medium of uniform temperature and thermal resistivity with an axial separation between cylinders of twice the actual depth of burial and with source and sink respectively generating and absorbing heat at identical rates, thereby resulting in the temperature of the horizontal midplane between cylinders (i.e., corresponding to the surface of the earth) remaining, by symmetry, undisturbed.

The principle of superposition, as applied to the case at hand, can be stated in thermal terms as follows: If the thermal network has more than one source of temperature rise, the heat that flows at any point, or the temperature drop between any two points, is the sum of the heat flows and temperature drops at these points which would exist if each source of temperature rise were considered separately. In the case at hand, the sources of heat flow and temperature rise to be superimposed are, namely, the heat $N$ refers to the number of cables or pipes, and $F$ is equal to unity when $N = 1$.

When the cable system is contained within a concrete envelope such as a duct bank, the effect of the differing thermal resistivity of the concrete envelope is conveniently handled by first assuming that the thermal resistivity of the medium is that of concrete $\beta_c$ through-out and then correcting that portion lying beyond the concrete envelope to the thermal resistivity of the earth $\beta_e$. Thus

$$ R_e' = 0.012 \beta_e n' \times \left[ \log \frac{D_s^t}{D_s^t + (LF) \log \left( \frac{4L}{D_s^t} F \right)} \right] + \frac{0.012 (\beta_e - \beta_n) n' (LF) G_0}{D_s^t} \text{ thermal ohm-feet} \hspace{1cm} (44A) $$

The empirical development of this equation is discussed in Appendix III. For a daily loss cycle and a representative value of $\alpha = 2.75$ square inches per hour for earth, $D_e$ is equal to 8.3 inches. It should be noted that the value of $D_e$ obtained from equation 45 is applicable for pipe diameters exceeding $D_e$, in which case the first term of equation 44 is negative.

The factor $F$ accounts for the mutual heating effect of the other cables of the cable system, and consists of the product of the ratios of the distance from the reference cable to the image of each of the other cables to the distance to that cable. Thus,

$$ F = \left( \frac{d_{41}^t}{d_{41}^t} \right) \left( \frac{d_{42}^t}{d_{42}^t} \right) \cdots \left( \frac{d_{4N}^t}{d_{4N}^t} \right) (N - 1 \text{ terms}) \hspace{1cm} (46) $$

It will be noted that the value of $F$ will vary depending upon which cable is selected as the reference, and the maximum conductor temperature will occur in the cable for which $4LF / D_e$ is a maximum. $N$ refers to the number of cables or pipes, and $F$ is equal to unity when $N = 1$.\(^\dagger\)

[Note: The text continues with further equations and discussion on thermal resistance and heat flow in cable systems, but is not fully transcribed here.]
from the cable, the outward flow of heat from the core of the earth, and the inward heat flow soil radiation, and, when present, the heat flow from interfering sources. By employing as the ambient temperature in the calculations the temperature at the depth of burial of the hottest cable, the combined heat flow from earth core and solar radiation sources is superimposed upon that produced at the surface of the hottest cable by the heat flow from that cable and interfering sources which are calculated separately with all other heat flows absent. The combined heat flow from earth core and solar sources results in an heat temperature which decreases with depth in summer; increases with depth in winter; remains about constant at any given depth on the average over a year; approximates constant at all depths at midseason, and in turn results in flow of heat from cable sources to earth's surface, directly to surface in midseason and winter and indirectly to surface in summer.

Factors which tend to invalidate the combined Kennelly-superposition principle as isothermal (as evidenced by melting of snow in winter directly over a buried steam main) and nonuniformity of thermal resistivity (due to such phenomena as radial and vertical migration of moisture). The extent to which the Kennelly-superposition principle method is invalidated, however, is not of practical importance provided that an over-all or effective thermal resistivity is employed in the Kennelly equation.

Special Conditions

Although the majority of cable temperature calculations may be made by the foregoing procedure, conditions frequently arise which require somewhat specialized treatment. Some of these are covered herein.

Emergency Ratings

Under emergency conditions it is frequently necessary to exceed the stated normal temperature limit of the conductor $T_e$ and set an emergency temperature limit $T'_e$. If the duration of the emergency is long enough for steady-state conditions to obtain, then the emergency rating $I'$ may be found by equation 9 substituting $T'_e$ for $T_e$ and correcting $R_{de}$ for the increased conductor temperature.

If the duration of the emergency is less than that required for steady-state conditions to obtain, the emergency rating of the line may be determined from

$$I' = \frac{T'_e - T_e}{R_{de}(1 + \gamma_e)(R_{int}' - R_{int})} - (T_a + \Delta T_a)$$

$kiloamperes (47)$

in which $R_{de}'$ is the effective transient thermal resistance of the cable system for the stated period of time. Procedures for calculating $R_{de}'$ for times up to several hours are given in reference 14, and for longer times in references 15-17.

The Effect of Extraneous Heat Sources

In the case of multicable installations the assumption has been made that all cables are of the same size and are similarly loaded. When this is not the case the temperature rise or load capability of one particular equal cable group may be determined by treating the heating effect of other cable groups separately, introducing an interference temperature rise $\Delta T_{int}$ in equations 1 and 9. Thus

$$T_e - T_a = \Delta T_e + \Delta T_d + \Delta T_{int}$$

degrees centigrade (1A)

$$I = \sqrt{\frac{T_e - (T_a + \Delta T_e + \Delta T_{int})}{R_{de}(1 + \gamma_e) R_{int}'}}$$

degrees centigrade (9A)

$kiloamperes (49)$

in which $\Delta T_{int}$ represents the sum of a number of interference effects, for each of which

$$\Delta T_{int} = \frac{\Delta T_{int} \cdot W_{de} (LF) + W_d}{R_{int}}$$

degrees centigrade (48)

$$R_{int} = 0.012 \beta_n' \log F_{int}$$

thermal ohm-feet (49)

$$F_{int} = \left(\frac{d_{int}(d_{int}(d_{int}(d_{int}(d_{int}))))}{d_{int}(d_{int}(d_{int}(d_{int}(d_{int}))))} \right) (N \text{ terms})$$

thermal ohm-feet (50)

where the parameters apply to each system which may be considered as a unit. For cables in duct

$$R_{int} = 0.012 n' \beta_n' \log F_{int} + N(\beta_n'-\beta_n)G_o$$

thermal ohm-feet (49A)

Because of the mutual heating between cable groups, the temperature rise of the interfering groups should be rechecked. If all the cable groups are to be given mutually compatible ratings, it is necessary to evaluate $W_e$ for each group by successive approximations, or by setting up a system of simultaneous equations, substituting for $W_e$ its value by equation 15 and solving for $I$.

In case $\Delta T_{int}$ or a component of it is produced by an adjacent steam main, the temperature of the steam $T_s$ rather than the heat flow from it is usually given. Thus

$$\Delta T_{int} = \left[ T_e - T_a \right] R_{int}$$

degrees centigrade (51)

where $R_{de}$ is the thermal resistance between the steam pipe and ambient earth.

Aerial Cables

In the case of aerial cables it may be desirable to consider both the effects of solar radiation which increases the temperature rise and the effect of the wind which decreases it. Under maximum sunlight conditions, a lead-sheathed cable will absorb about 4.3 watts per foot per inch of profile which must be returned to the atmosphere through the thermal resistance $R_s/n'$. This effect is conveniently treated as an interference temperature rise according to the relationship

$$\Delta T_{int} = 4.3 D_s / R_s / n'$$

degrees centigrade (47A)

For black surfaces this value should be increased about 75%.

As indicated in Appendix II, the following expression for $R_s$ may be used where $V_s$ is the velocity of the wind in miles per hour

$$R_s = \frac{3.5 n'}{D_s \left(\sqrt{V_s / D_s} + 0.62 \right)}$$

thermal ohm-feet (47B)

Use of Low-Resistivity Backfill

In cases where the thermal resistivity of the earth is excessively high, the value of $R_s$ may be reduced by backfilling the trench with soil or sand having a lower value of thermal resistivity. Equation 44(A) may be used for this case if $R_s$, the thermal resistivity of the backfill is substituted for $R_s$ and $G_s$ applies to the zone having the backfill in place of the zone occupied by the concrete.

Single-Conductor Cables in Duct with Solidly Bonded Sheaths

The relatively large and unequal sheath losses in the three phases which may result from this type of operation may be determined from Table VI of reference 1. It will be noted that

$$Y_{ret} = \frac{R_s}{R_{de}} \left(\frac{I_{in}^2}{I^2} \right)$$

$$Y_{ret} = \frac{R_s}{R_{de}} \left(\frac{I_{in}^2}{I^2} \right)$$

$$Y_{ret} = \frac{R_s}{R_{de}} \left(\frac{I_{in}^2}{I^2} \right)$$

$$Y_{ret} = \frac{R_s}{R_{de}} \left(\frac{I_{in}^2}{I^2} \right)$$

$$Y_{ret} = \frac{R_s}{R_{de}} \left(\frac{I_{in}^2}{I^2} \right)$$

where expressions for $I_{in}^2/I^2$ etc., appear in the table. The resulting unequal values of $Y_{ret}$ in the three phases will yield unequal values of $q_a$ and equation 5 becomes for phase no. 1, the instance given as equation 5(A) on the following page.


**Table VIII. Constants for Use in Equation 53**

<table>
<thead>
<tr>
<th>Condition</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>Average AT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cable in metallic conduit</td>
<td>0.07</td>
<td>0.121</td>
<td>0.0017</td>
<td>20</td>
</tr>
<tr>
<td>Cable in fiber duct in air</td>
<td>0.07</td>
<td>0.036</td>
<td>0.0009</td>
<td>20</td>
</tr>
<tr>
<td>Cable in fiber duct in concrete</td>
<td>0.07</td>
<td>0.043</td>
<td>0.0014</td>
<td>20</td>
</tr>
<tr>
<td>Cable in transite duct in air</td>
<td>0.07</td>
<td>0.0086</td>
<td>0.0008</td>
<td>20</td>
</tr>
<tr>
<td>Cable in transite duct in concrete</td>
<td>0.07</td>
<td>0.079</td>
<td>0.0016</td>
<td>20</td>
</tr>
<tr>
<td>Gas-filled pipe-type cable at 200 psi</td>
<td>0.07</td>
<td>0.121</td>
<td>0.0017</td>
<td>10</td>
</tr>
</tbody>
</table>

where $\eta$ is the average of $g_{ah}$, $g_{an}$, and $g_{ah}$.

**Armed Cables**

In multiconductor armored cables a loss occurs in the armor which may be considered as an alternate to the conduit or pipe loss. If the armor is nonmagnetic, the component of armor loss $Y_2$ to be used instead of $Y_2$ in equations 14 and 19 may be calculated by the equations for sheet loss substituting the resistance and mean diameter of the armor for those of the sheath. In calculating the armor resistance, account should be taken of the spiralling effect for which equation 13 suitably modified may be used. If the armor is magnetic, one would expect an increase in the factors $Y_2$ and $Y_3$ in equation 14 since this occurs in the case of magnetic conduit. Unfortunately, no simple procedure is available for calculating these effects. A rough estimate of the inductive effects may be made by using the procedure given above for magnetic conduit. A simple method of approximating the losses in single conductor cables with steelwire armor at spacings ordinarily employed in submarine installations is to assume that the combined sheath and armor current is equal to the conductor current. The effective a-c resistance of the armor may be taken as 30 to 60% greater than its d-c resistance corrected for lay as indicated above. If more accurate calculations are desired, references 19 and 20 will be found useful.

**Effect of Forced Cooling**

The temperature rise of cables in pipes or tunnels may be reduced by forcing air axially along the system. Similarly, in the case of oil-filled pipe cable, oil may be circulated through the pipe. Under these conditions, the temperature rise is not uniform along the cable and increases in the direction of flow of the cooling medium. The solution of this problem is discussed in reference 21.

**Appendix I**

**Development of Equations 41, 42, and Table VII**

Theoretical and semiempirical expressions for the thermal resistance between cables for the case of an enclosing pipe or duct wall are given in reference 10. Further data on the thermal resistance between cables and fiber and transite ducts are given in reference 11. For purposes of cable rating, it is desirable to develop standardized expressions for these thermal resistances based upon all of the data available and including the effect of the temperature of the intervening medium.

The theoretical expression for the case where the intervening medium is air or gas as presented in reference 10 may be generalized in the following form:

$$R_{ad} = \frac{n'}{D_x \left[ \frac{\Delta T P_{1/4}^{D_x}}{D_x} + b + c T_m \right]}$$  \hspace{1cm} (53)

where $R_{ad}$ = the effective thermal resistance between cable and enclosure in thermal ohm-feet.

$D_x$ = the cable diameter or equivalent diameter of three cables in inches.

$\Delta T$ = the temperature differential in degrees centigrade.

$P$ = the pressure in atmospheres.

$T_m$ = mean temperature of the medium in degrees centigrade.

$n'$ = number of conductors involved.

The constants $a$, $b$, and $c$ in this equation have been established empirically as follows: Considering $b + c T_m$ as a constant for the moment, the analysis given in reference 10 results in a value of $a = 0.07$. With $a$ thus established, the data given in reference 10 for cable in pipe, and in reference 11 for cable in fiber and transite ducts were analyzed in similar manner to give the values of $b$ and $c$ which are shown in Table VIII. In order to avoid a retaring calculation procedure, it is desirable to assume a value for $\Delta T$ since its actual value will depend upon $R_{ad}$ and the heat flow. Fortunately, as $\Delta T$ occurs to the $1/4$ power in equation 53, the use of an average value as indicated in Table VIII will not introduce a serious error.

By further restricting the range of $D_x$ to 1–4 inches for cable in duct or conduit and to 3–5 inches for pipe-type cables, equation 53 is reduced to equation 41.

$$R_{ad} = \frac{n'}{1 + (2 + C T_m) D_x^{1/4}}$$  \hspace{1cm} (41)

in which the values of the constants $A$, $B$, and $C$ appear in Table VII. In the case of oil-filled pipe cable, the analysis given in reference 10 gives the following expression:

$$R_{ad} = 0.60 + 0.025(D_x^{1/4} T_m^{1/2} AT)^{1/4}$$  \hspace{1cm} (54)

thermal ohm-feet

Assuming an average value of $\Delta T = 7$ C and a range of 150–350 for $D_x T_m$, equation 54 reduces to equation 41 with the values of $A$, $B$, and $C$ given in Table VII.

In the case of cables or pipes suspended in still air, the heat loss by radiation may be determined by the Stefan-Boltzmann formula:

$$n' W(\text{radiation}) = 0.139 D_x^{1/4} \left( T_m^{273} - (T_x^{273}) \right) 10^{-4}$$  \hspace{1cm} (55)

watts per foot

where $\epsilon$ is the coefficient of emissivity of the cable or pipe surface. Over the limited temperature range in which we are interested, equation 55 may be simplified to:

$$n' W(\text{radiation}) = 0.102 D_x^{1/4} T_m^{273}$$  \hspace{1cm} (55a)

watts per foot

Over the same temperature range the heat loss by convection from horizontal cables or pipes is given with sufficient accuracy by the expression:

$$n' W(\text{convection}) = 0.064 D_x^{1/4} T_m^{273}$$  \hspace{1cm} (56)

watts per foot

in which the numerical constant 0.064 has been selected for the best fit with the carefully determined test results reported by Heilman on 1.3, 3.5 and 10.8-inch diameter black pipes ($\epsilon = 0.95$). Incidentally, this value also represents the best fit with the test data on 1.9-4.5 inch diameter black pipes reported by Rosch.[31]

For vertical cables or pipes the value of this numerical constant may be increased by 25%.[31]

Combining equations 55(A) and 56(A) with $T_m = 45$ C we obtain the relationship:

$$\bar{R}_d = \frac{\Delta T}{n' W(\text{total})}$$  \hspace{1cm} (57)

$$= \frac{15.6 n'}{D_x^{1/4} T_m^{273} + 1.96 \left( D_x^{1/4} T_m^{273} \right)^{1/4} + 1.96 \left( 1 + 0.0167 T_m \right)}$$  \hspace{1cm} (58a)

thermal ohm-feet

If the cable is subjected to wind having a velocity of $V_w$ miles per hour, the following expression derived from the work of Schurig and Frick[31] should be substituted for the convection component:

$$n' W(\text{convection}) = 0.286 D_x^{1/4} T_m^{273} V_w/D_x$$  \hspace{1cm} (56a)

watts per foot

Combining equations 55(A) and 56(A) with $T_m = 45$ C we obtain:

$$\bar{R}_d = \frac{\Delta T}{n' W(\text{total})}$$  \hspace{1cm} (57)

$$= \frac{3.5 n'}{D_x^{1/4} T_m^{273} + 0.824}$$  \hspace{1cm} (42a)

thermal ohm-feet

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Appendix II

Determination of the Geometric Factor G₀ for Duct Bank

Considering the surface of the duct bank to act as an isothermal circle of radius \( r_s \), the thermal resistance between the duct bank and the earth's surface will be a logarithmic function of \( r_s \) and \( L_b \) the distance of the center of the bank below the surface. Using the long form of the Kennelly Formula¹⁸ we may define the geometric factor \( G₀ \) as

\[
G₀ = \log \frac{L_b}{r_s} + \sqrt{L_b^2 - r_s^2}
\]

\[
= \log \left[ \frac{L_b}{r_s} + \sqrt{\frac{L_b^2}{r_s^2} - 1} \right] - 1
\]

(57)

In order to evaluate \( r_s \) in terms of the dimensions of a rectangular duct bank, let the smaller dimension of the bank be \( x \) and the larger dimension by \( y \). The radius of a circle inscribed within the duct bank touching the sides is

\( r = \frac{x}{2} \)

(58)

and the radius of a larger circle embracing the four corners is

\[
r_s = \frac{\sqrt{x^2 + y^2}}{2}
\]

(59)

Let us assume that the circle of radius \( r_s \) lies between these circles and the magnitude of \( r_s \) is such that it divides the thermal resistance between \( r_1 \) and \( r_2 \) in direct relation to the portions of the heat field between \( r_1 \) and \( r_2 \) occupied and unoccupied by the duct bank. Thus

\[
\log \frac{r_s}{r_1} = \frac{xy - xy^2}{r(s^2 - r_1^2)} \log \frac{r_1}{r_s}
\]

or

\[
\log \frac{r_s}{r_2} = \frac{\pi r_2^2 - xy}{r(s^2 - r_2^2)} \log \frac{r_2}{r_s}
\]

(60)

from which

\[
\log \frac{r_s}{r_2} = -\frac{1}{2} \log \left( \frac{1 + x/y}{1 + y/x} \right) + \log \frac{x}{y}
\]

(61)

It is desirable to derive \( r_s \) in terms of the perimeter \( P \) of the duct bank. Thus

\[
P = 2(x + y) = x + y/2
\]

and therefore

\[
\log \frac{r_s}{P^2} = -\log \frac{4}{1 + y/x}
\]

(62)

The curves of Fig. 2 have been developed from equations 57, 60, and 61 for several values of the ratio \( y/x \). It should be noted in passing that the value of \( r_s = 0.112^{2} \) used in reference 13 applies to a \( y/x \) ratio of about 2.1 only.

Appendix III

Empirical Evaluation of \( D_s \)

In order to evaluate the effect of a cyclic load upon the maximum temperature rise of a cable system simply, it is customary to assume that the heat flow in the final

\[
D_s = 8.3 \text{ inches. As indicated in the third paper of reference 3, however, theoretically } D_s \text{ should vary as the square root of the product of the diffusivity and the time length of the loading cycle. Hence as the diffusivity was taken as 2.75 square inches per hour in the above,}
\]

\[
D_s = 1.02 \times \sqrt{\alpha \times \text{length of cycle in hours}}
\]

(45)

Table IX presents a comparison of the values of per cent attainment factor for sinusoidal loss cycles at 30\% loss factor as calculated by equations 45, 60, 62(A), and 63 and as they appear in Table II of the first paper of reference 3.

Appendix IV. Calculations for Representative Cable Systems

15-KV 350-MCM—3-Conductor Shielded Compact Sector Paper and Lead Cable Suspended in Air

\( D_s = 0.616 \) (equivalent round); \( V = \text{gauge depth} = 0.559 \text{ inch} \)

\( D_s = 2.129; T = 0.175 \text{ inch}; t = 0.120 \text{ inch} \)

\( T_s = 81 \text{ C}; R_{dc} = 12.9 \times 10^{-8} \text{ (234.5 + 81)} = 0.350 \text{ (234.5 + 75)} = 37.6 \text{ microhms per foot (Eq. 10A)} \)

\( D_{es} = 2.129 - 0.120 = 2.009 \text{ inches (Eq. 12)} \)

\( R_s = 37.9 \times 10^{-6} = 157 \text{ microhms per foot at 50 C (Eq. 11A)} \)

\( k_s = 1.0; k_p = 0.6 \) (equivalent round) (Table II)

\( R_{dc}/k_s = 37.6; Y_{es} = 0.008 \) (Eq. 21 and Fig. 1)

\( S = 0.616 + 2(0.175 + 0.008) = 0.982 \text{ inches} \)

\( R_{dc}/k_p = 62.6; F_s(x) = 0.003 \) (Fig. 1)

\( Y_{cp} = 1.2 \times 0.016 \times 0.982 \times 0.003 = 0.002 \) (Eq. 24A. and note to Table II)

\( 1 + T_s = 1 + 0.008 + 0.002 = 1.010 \)

\( t_s = 1.55(0.175 + 0.008) + 0.60(0.539) = 0.534 \text{ inch (Eq. 32)} \)

\( Y_s = Y_{es} - \frac{356}{157(37.6)} = 0.019 \) (Eq. 31A)

\( R_{dc}/R_{es} = 1.010 + 0.019 = 1.029 \) (Eq. 14)

\( q_s = q_e + 1 + 0.019 = 1.019 \) (Eqs. 18-19)

\( \cos \phi = 3.7 \text{ (Table V); } E = 15/\sqrt{3} = 8.7; \cos \phi = 0.022 \)

\( W_s = 2(1.575 + 0.051) \times 0.081 = 0.094 \text{ watt per conductor foot} \) (Eq. 36 and text)

(Note: In computing dielectric loss on

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sector conductors, the equivalent diameter of the conductor is taken equal to that of a concentric round conductor, i.e., 0.681 inch for 350 MCM.)

\[ \dot{J} = 700 \text{ (Table VI); } G_i = 0.45 \]

(Table VIII of reference 1)

\[ R_{e1} = \frac{0.00522\{700(0.45)\}}{1.64} \]

thermal ohm-feet (Eq. 39)

\[ n' = 3; \quad \epsilon = 0.41 \text{ (assumed)} \]

\[ R_e = \frac{9.5(3)}{1+1.7[2.129(0.41+0.41)]} = 7.18 \text{ thermal ohm-feet (Eq. 42A)} \]

\[ R_{eo} = 1.64 + 1.019(7.18) = 8.96 \text{ thermal ohm-feet (Eq. 8)} \]

\[ \Delta T_{e1} = 0.094(0.82+7.18) = 0.75 \text{ C (Eq. 5)} \]

\[ T_a = 40 \text{ C (assumed)} \]

\[ I = \sqrt{\frac{81 - (40+0.8)}{36.7(1.010(8.96))}} = 0.344 \text{ kiloamperes (Eq. 9)} \]

If the cable is outdoors in sunlight and subjected to an 0.84 mile per hour wind

\[ R_e = \frac{3.5(3)}{2.129(\sqrt{0.84/2.129} + 0.62(0.41))} = 5.59 \text{ thermal ohm-feet (Eq. 42B)} \]

\[ R_{eo} = 1.64 + 1.019(5.59) = 7.34 \text{ thermal ohm-feet (Eq. 8)} \]

\[ \Delta T_{e1} = (4.3)(2.129) \left( \frac{5.59}{3} \right) = 17.1 \text{ C (Eq. 47A)} \]

\[ T_a = 30 \text{ C (assumed)} \]

\[ I = \sqrt{\frac{81 - (30+0.6+17.1)}{(37.6)(1.010)(7.34)}} = 0.346 \text{ kiloamperes (Eq. 9)} \]

In this particular case the net effect of solar radiation and an 0.84 mile per hour wind is to effectively raise the ambient temperature by 10 degrees, which is a rough estimating value commonly used. It should be noted, however, that this will not always be true, and the procedure outlined above is preferable.

69-KV 1,500-MCM—Single-Circuit Conductor Oil-Filled Cable in Duct

Two identical cable circuits will be considered in a 2 by 3 fiber and concrete duct structure having the dimensions shown in Fig. 3.

\[ D_o = 0.600; \quad D_r = 1.543; \quad D_i = 2.113; \quad T = 0.285; \quad D_f = 2.733; \quad t = 0.130 \text{ inches} \]

\[ T_e = 75 \text{ C; } R_{e1} = \frac{12.9}{1.50} = 8.60 \text{ microhms per foot (Eq. 10A)} \]

\[ D_{sm} = 2.373 - 0.130 = 2.243 \text{ inches (Eq. 12)} \]

\[ \frac{R_s}{(2.243)(0.130)} = 37.9 \text{ microhms per foot at 50 C (Eq. 11A)} \]

\[ k_t = 1.543 - 0.600 = 1.543 - 1.200 = 0.340 \text{ (Eq. 23 and Table II)} \]

\[ k_p = 0.72; \quad k_p = 0.8 \]

\[ Y_{e1} = 0.075; \quad Y_{e2} = 0.075 \text{ (Eq. 21 and Fig. 1)} \]

\[ S = 9.0 \text{ (Fig. 3); } R_{e1}/k_p = 10.75; \quad F(x_p') = 0.075 \text{ (Fig. 1)} \]

\[ R_{e1}/R_{e2} = 1.082 + 0.006 = 1.088 \text{ (Eq. 14)} \]

\[ Q_1 = Q_2 = 1 + 0.006 = 1.006 \text{ (Eqs. 18-19)} \]

\[ e = (\text{Table V}); \quad E = 60/\sqrt{3} = 40; \quad \cos \phi = 0.005 \]

\[ W_d = 0.00276(40)(3.5)(0.005) \]

\[ \frac{1+\frac{5}{12}(\frac{2.243}{2(9.0)})^3}{1+\frac{5}{12}(\frac{2.243}{2(9.0)})^3} = 0.006 \text{ (Eq. 30A)} \]

\[ R_{e1}/R_{e2} = 1.082 + 0.006 = 1.088 \]

\[ 1 + Y_e = 1 + 0.075 + 0.007 = 1.082 \]

Assuming the sheaths to be open-circuited, \( Y_{e1} = 0 \)

\[ Y_e = Y_{e2} = \frac{396}{130(8.60) \frac{2.243}{2(9.0)}^3} \times \]

\[ \log \frac{2.113}{1.543} = 0.57 \text{ watt per conductor foot (Eq. 36)} \]

\[ \text{Fig. 3. Assumed duct bank configuration for typical calculations on 69-kv 1,500-MCM oil-filled cable (Appendix IV)} \]
\[ \Delta T_d = 0.57 (0.45 + 1.75 + 0.24 + 4.63) = 9.40 \text{ C} \]  
\[ W_{cf} = (1.17)(8.60)(1.082) = 9.31 I_d^2 \text{ watts per conductor foot} \]  
\[ \Delta T_{int} = 9.31 I_d (1.000)(0.80) + 0.57 = 2.17 + 28.5 I_d^2 \text{ degrees centigrade in circuit no. 2} \]  

Similar calculations for the second circuit yield the following values.

\[ \Delta T_{o} = 7.18; \Delta T_d = 3.4; W_{ca} = 17.44 I_d^2; \]  
\[ \Delta T_{int} = 1.71 + 53.2 I_d^2 \text{ in circuit no. 1} \]  

Solving simultaneously \[ I_1 = 0.714; I_2 = 0.487 \text{ kiloamperes.} \]

138-KV 2,000-MCM High-Pressure Oil-Filled Pipe-Type Cable 8.625-Inch-Inch-Diameter Pipe

The cable shielding will consist of an intercalated 7/8(0.003)-inch bronze tape—1-inch lay, and a single 0.1(0.2)-inch D-shaped brass skin wire—1.5-inch lay. The cables will lie in cradled configuration.

\[ D_0 = 1.632; D_1 = 2.642; T = 0.505; \]  
\[ D_2 = 2.661; D_3 = 8.125 \]

\[ T_o = 70 \text{ C}; R_{ca} = 12.9 \times \frac{234.5 + 70}{234.5 + 75} = 6.35 \text{ microamps per foot} \]  
\[ \Delta T_d = 0.001082(0.003) = 0.000263; \]  
\[ I = 1.0; \rho = 23.8; \]  
\[ N = 54.4 \]

For shield wire \[ A_s = 7/8(0.003) = 0.00263; \]  
\[ I = 1.0; \rho = 23.8; \]  
\[ R_s = 23.8 \times \frac{1}{4(0.00263)} \]

\[ \frac{564 + 50}{564 + 20} = 62,900 \text{ microamps per foot at 50 C} \]  

For skid wire \[ A_s = \frac{1}{2} \pi(0.1)^2 = 0.0157; \]  
\[ I = 1.0; \rho = 38; \]  
\[ R_s = 38 \times \frac{1}{4(0.00157)} \]

\[ \frac{912 + 50}{912 + 20} = 11,100 \text{ microamps per foot at 50 C} \]  

\[ R_s (\text{net}) = \left( \frac{829(11.1)}{829(11.1)} \right) 1,000 = 9,453 \text{ microamps per foot at 50 C} \]  

\[ k_s = 0.435; k_p = 0.37 \]  
\[ \frac{R_d}{k_s} = 14.6; Y_{ca} = 0.052(1.7) = 0.088 \]  
\[ (E, 21, \text{ Fig. 1, and text}) \]

\[ S = 2.66 + 0.10 = 2.76; \]  
\[ R_{ca} = 19.4 \]  
\[ F_{(x_p')} = 0.035 \]  

\[ \Delta T_d = 1.88 + 1.009(0.77) = 1.327(0.17 + 2.85) = 0.17 \text{ thermal ohm-foot} \]  

\[ \Delta T_d = 1.48(0.69 + 0.77 + 0.17 + 3.38) = 7.4 \text{ C} \]  

\[ T_o = 25 \text{ C (assumed);} \]

\[ I = \sqrt{70 - 25 + 1.74 + 24 + 3.74} = 6.65 \text{ thermal ohm feet (Eq. 83)} \]

\[ \Delta T_d = 0.57 (0.45 + 1.75 + 0.24 + 4.63) = 9.40 \text{ C} \]  

\[ W_{cf} = (1.17)(8.60)(1.082) = 9.31 I_d^2 \text{ watts per conductor foot} \]  

\[ \Delta T_{int} = 9.31 I_d (1.000)(0.80) + 0.57 = 2.17 + 28.5 I_d^2 \text{ degrees centigrade in circuit no. 2} \]  

\[ Y_{ca} = 20.0 \text{ microamps per foot} (\text{Eq. 29A}) \]

\[ Y_{ca} = 0.052(1.7) = 0.088 \]  

\[ (E, 21, \text{ Fig. 1, and text}) \]

\[ S = 2.66 + 0.10 = 2.76; \]  
\[ R_{ca} = 19.4 \]  
\[ F_{(x_p')} = 0.035 \]  

\[ \Delta T_d = 1.88 + 1.009(0.77) = 1.327(0.17 + 2.85) = 0.17 \text{ thermal ohm-foot} \]  

\[ \Delta T_d = 1.48(0.69 + 0.77 + 0.17 + 3.38) = 7.4 \text{ C} \]  

\[ T_o = 25 \text{ C (assumed);} \]

\[ I = \sqrt{70 - 25 + 1.74 + 24 + 3.74} = 6.65 \text{ thermal ohm feet (Eq. 83)} \]